Tanta university
Faculty of engineering
Computer and automatic control department

Image processing course
Fourth year students
Sheet 4, Date : 26/10/2011

Sheet 4

1. Prove that finding the maximum intensity value in a digital image is a nonlinear operation

2. Image filtering using the median of the pixel neighborhood is useful in removing certain kind of noise. The median, ζ , of a set of numbers is such that half the values in the set are below ζ and the other half are above it. For example, the median of the set of values {2, 3, 8, 20, 21, 25, 31} is 20.
   a) Show that an operator that computes the median of a sub-image area, S, is nonlinear.
   b) Is it possible to apply the median on images using a mask and the correlation? Why?

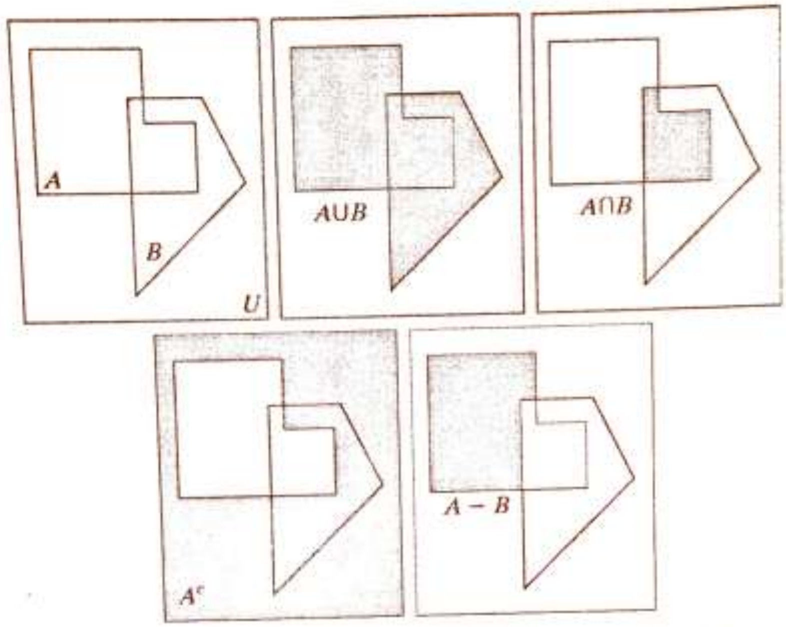
3. Imaging under very low light levels causes the imaging system sensor noise largely affects the resulting images. Let $g(x, y)$ denote a corrupted image formed by the addition of sensor noise: $\eta(x, y)$ to the scene noiseless image $f(x, y)$; that is $g(x, y) = f(x, y) + \eta(x, y)$. In a process of noise reduction, a set of noisy images $\{g_i(x, y)\}$ for the same scene are averaged to get a less affected by noise image. Assume that at every pair of coordinates (x, y) the noise in the added images is uncorrelated and has zero average value.
   a) Prove that the expected value of the image formed by averaging K different noisy images is the noiseless image $f(x, y)$
   b) How the variance and the standard deviation of the average image is related to the variance and the standard deviation, respectively, of the noise.
   c) Mention the condition that must be satisfied when practically applying the process for reducing the noise

4. Image subtraction is used often in industrial applications for detecting missing components in product assembly. The approach is to store a "golden" image that corresponds to a correct assembly; this image is then subtracted from incoming images of the same product. Ideally, the difference would be zero if the new products are assembled correctly. Difference images for products with missing components would be non-zero in the area where they are differs from the golden image. What conditions do you think have to be met in practice for this method to work?

Report 5. Consider two 8 bit images whose intensity levels span the full range from 0 to 255. Discuss the limiting effect of repeatedly subtracting image 2 from image 1. Assume that the result also is represented in 8 bits

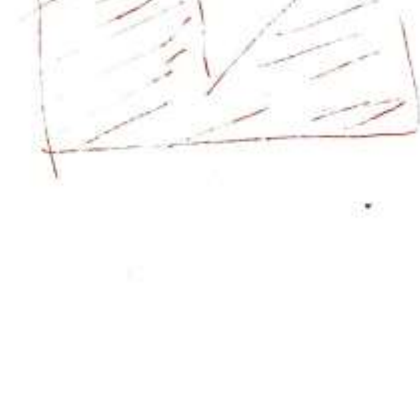
6. For the figure shown, sketch the set $A \cap B) \cup (A \cup B)^c$

the process of repeated Subtraction of image
$d_K(x,y) = a(x,y) - \sum_{k=1}^{K} b(x,y)$
results a differenca whos components



Report 7.

a) Discuss one application of image addition → Noise reduction by averaging
b) Discuss one application for image subtraction → Enhancement of differences between images.



Sheet # 4

① Prove that finding the maximum intensity value in a digital image is a nonlinear operation?

An operation H on an image $f(x,y)$ is linear if and only if it satisfies two conditions:

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① $H[a f(x,y)] = a H[f(x,y)]$ (homogeneity)

② $H[a_i f_i(x,y) + a_j f_j(x,y)] =$
$a_i H[f_i(x,y)] + a_j H[f_j(x,y)]$ (Additivety)

Example for the maximum pixel value operation:

$$Max\left\{(1)\underbrace{\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}}_{f_i(x,y)} + (-1)\underbrace{\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}}_{f_j(x,y)}\right\}$$

let $a_i = 1$
$a_j = -1$

$$= Max\left\{\begin{bmatrix} -6 & -3 \\ -2 & -4 \end{bmatrix}\right\} = -2 \rightarrow LHS$$

$$(1)\,Max\left\{\begin{bmatrix} 0 & 2 \\ 2 & 3 \end{bmatrix}\right\} + (-1)\,Max\left\{\begin{bmatrix} 6 & 5 \\ 4 & 7 \end{bmatrix}\right\}$$

$$= 3 - 7 = -4 \rightarrow RHS$$

$LHS \neq RHS$ ∴ the maximum operation is non linear.

(2) Image filtering using the median of the pixel neighborhood is useful in removing certain kind of noise. The median ζ of a set of numbers is such that half the values in the set are below ζ and the other half are above it. For example, the median of the set of values $\{2, 3, 8, 20, 21, 25, 31\}$ is 20.

a) Show that an operator that computes the median of a sub-image area, S, is non linear.

The median of a sub-image is a linear operation if and only if

$$\text{Median}[a_1 f_1(x,y) + a_2 f_2(x,y)] = a_1 \text{Median}[f_1(x,y)] + a_2 \text{Median}[f_2(x,y)] \rightarrow (1)$$

for any real constants a_1 & a_2 where f_1 and f_2 are any two sub-images.

Let $f_1 = \begin{bmatrix} 3 & 4 & 3 \\ 4 & 8 & 7 \\ 9 & 8 & 9 \end{bmatrix}$ and $f_2 = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 3 & 1 \\ 2 & 0 & 0 \end{bmatrix}$

$\text{Median}[f_1(x,y)] = 7$

$\text{Median}[f_2(x,y)] = 1$ Let: $a_1 = 1$ & $a_2 = -1$

LHS of (1)

$a_1 \text{Median}[f_1(x,y)] + a_2 \text{Median}[f_2(x,y)] = 7 - 1 = 6$

RHS of (1)

$\text{Median}[a_1 f_1(x,y) + a_2 f_2(x,y)] = \text{Median}\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$

$= 5 \neq \text{LHS}$

So, the median operation is not linear

For the memories of yesterday,
For the happiness of today...

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b) Is it possible to apply the median on images using a mask and the Correlation ? why?

No, A mask and Correlation is used in linear filtering only.

③ Imaging under very low light levels, Causes the imaging system sensor noise Largely affects the resulting images. Let $g(x,y)$ denote a corrupted image formed by the addition of sensor nois: $\eta(x,y)$ to the scene noiseless image $f(x,y)$; that is

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$$g(x,y) = f(x,y) + \eta(x,y)$$

In a process of noise reduction, a set of noisy images $\{g_i(x,y)\}$ for the same scene are averaged to get a less affected by noise image. Assume that at every pair of Coordinates (x,y) the noise in the added images is uncorrelated and has zero average value.

a) Prove that the expected value of the image formed by averaging k different noisy images is the noisless image $f(x,y)$ → Prove that $E\{\bar{g}(x,y)\} = f(x,y)$

The average at any pair of Coordinates (x,y) of a set of k images is given by:

$$\bar{g}(x,y) = \frac{1}{k}\sum_{i=1}^{k} g_i(x,y)$$

$$= \frac{1}{k}\sum_{i=1}^{k} f_i(x,y) + \frac{1}{k}\sum_{i=1}^{k} \eta_i(x,y)$$

$g = f$?

$\bar{g} = \frac{1}{k} \sum (-)$

The expected value of the image $\bar{g}(x,y)$:

$$E\{\bar{g}(x,y)\} = E\left\{\frac{1}{k}\sum_{i=1}^{k} f_i(x,y) + \frac{1}{k}\sum_{i=1}^{k} \eta_i(x,y)\right\}$$

∵ Appling that the expected value of the sum is the same as the sum of the expected values of the items included in the sum.

$$\therefore E\{\bar{g}(x,y)\} = \frac{1}{k}\sum_{i=1}^{k} \underbrace{E\{f_i(x,y)\}}_{\text{expected value of a constant is the constant itself.}} + \frac{1}{k}\sum_{i=1}^{k} \underbrace{E\{\eta_i(x,y)\}}_{\text{expected value of a random variable of a given mean is the mean itself.}}$$

لأن $f_i(x,y)$ هي الصورة الأصلية ولم تتغير

$\eta_i(x,y)$ has zero mean (given)

$$\therefore E\{\bar{g}(x,y)\} = \frac{1}{k}\sum_{i=1}^{k} f_i(x,y) + \text{zero} = \frac{\cancel{k}}{\cancel{k}} f(x,y)$$

$$E\{\bar{g}(x,y)\} = f(x,y)$$

#

Don't worry.
Be happy

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b) How the variance and the standard deviation of the average image is related to the variance and the standard deviation, respectively, of the noise.

↳ Prove that $\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$

and $\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$

$\because \bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i(x,y)$

$$= \frac{1}{K}\sum_{i=1}^{K} f_i(x,y) + \frac{1}{K}\sum_{i=1}^{K}\eta_i(x,y)$$

- The variance of $\bar{g}(x,y)$:

$$\sigma^2_{\bar{g}(x,y)} = \sigma^2\left(\frac{1}{K}\sum_{i=1}^{K} f_i(x,y)\right) + \sigma^2\left(\frac{1}{K}\sum_{i=1}^{K}\eta_i(x,y)\right)$$

From the random variable theory that the variance of the sum of uncorrelated random variables is the sum of the variances of those variables. and the variance of a constant value $f_i(x,y)$ is zero, then:

$$\therefore \sigma^2_{\bar{g}(x,y)} = \underbrace{\frac{1}{K^2}\sigma^2_{f(x,y)}}_{zero} + \frac{1}{K^2}\underbrace{\left[\sigma^2_{\eta_1}(x,y) + \dots + \sigma^2_{\eta_K}(x,y)\right]}_{K \text{ عددهم}}$$

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K^2}\left[\sigma^2_{\eta_1}(x,y) + \sigma^2_{\eta_2}(x,y) + \dots + \sigma^2_{\eta_K}(x,y)\right]$$

- But η_i are samples of the noise η that has a single variance $\sigma^2_{\eta(x,y)}$

$$\therefore \sigma^2_{\bar{g}(x,y)} = \frac{K}{K^2}\sigma^2_{\eta(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$

$$\sigma^2_{\bar{g}(x,y)} = \frac{1}{K}\sigma^2_{\eta(x,y)}$$ #

- The standard deviation is obtained by taking the square root of both sides in the above equation:

$$\boxed{\sigma_{\bar{g}(x,y)} = \frac{1}{\sqrt{K}}\,\sigma_{\eta(x,y)}}\ \#$$

c) Mention the condition that must be satisfied when practically applying the process for reducing the noise.

Because $E\{\bar{g}(x,y)\} = f(x,y)$, this means that $\bar{g}(x,y)$ approaches $f(x,y)$ as the number of noisy images used in the averaging process increases. So, the Images $g_i(x,y)$ must be registered (aligned) in order to avoid the introduction of blurring ضبابية and other artifacts in the output image.

④ Image subtraction is used often in industrial applications for detecting missing components in product assembly. The approach is to store a "golden" image that corresponds to a correct assembly. this image is then subtracted from incoming images of the same product. Ideally, the difference would be 0 if the new products are assembled correctly. Difference images for products with missing components would be non zero in the area where they are differs from the golded image. What Conditions do you think have to be met in practice for this method to work?

Let $g(x,y)$ ⟶ the golden image.
$f(x,y)$ ⟶ any input image acquired during routine operation of the system.

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The simple difference $d(x,y)$ is:

$$d(x,y) = g(x,y) - f(x,y)$$

The resulting image $d(x,y)$ can be used in two ways:

① Pixel-by-Pixel analysis:

In this case we say that $f(x,y)$ is "close enough" to the golden image if all the pixels in $d(x,y)$ fall within a specified threshold band $[T_{min}, T_{max}]$.

negative
positive } they are usually the same value

∴ We have a band $[-T, T]$ in which all pixels of $d(x,y)$ must fall in order for $f(x,y)$ to be declared acceptable.

② The second major approach is simply to sum all the pixels in $|d(x,y)|$ and compare the sum against a threshold Q. (we use absolute values).

There are three fundamental factors:

① Proper registeration: تعديل وضع الصورة لتتوافق مع وضع الصورة الذهبية.

② Controlled illumination: التحكم في الإضاءة لتتوافق مع إضاءة " "

③ Noise levels: ∴ كلما قل مستوى الـ noise على الصورة المدققة كلما كانت نتيجة تقاربها مع الصورة الذهبية أعلى

(40)

Close my eyes only for
a moment,and the moment's gone

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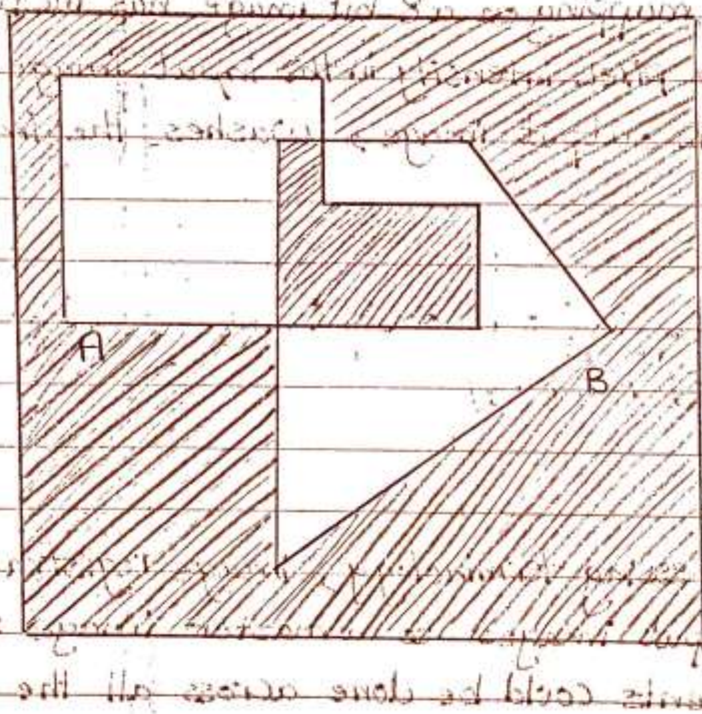
(5) Report

(6) For the figure shown, sketch the set

$(A \cap B) \cup (A \cup B)^C$



(7) Report

Sheet 4 solution

1. An operation H on a digital image f(x, y) is linear if and only if it satisfies two conditions:

$$H[af(x,y)] = aH[f(x,y)] \text{(homogeneity),and}$$

$$H[a_i f_i(x,y) + a_j f_j(x,y)] = a_i H[f_i(x,y)] + a_j H[f_j(x,y] \text{(superposition)}$$

Then one example that demonstrates that one of these conditions is not applicable for a given operation is sufficient as a proof for the nonlinearity of the operation. Here is an example that demonstrates the violation of the superposition for the maximum pixel value operation.

$$Max\left\{(1)\begin{bmatrix}0 & 2\\2 & 3\end{bmatrix} + (-1)\begin{bmatrix}6 & 5\\4 & 7\end{bmatrix}\right\}$$

$$= Max\left\{\begin{bmatrix}-6 & -3\\-2 & -4\end{bmatrix}\right\} = -2$$

while

$$(1)Max\left\{\begin{bmatrix}0 & 2\\2 & 3\end{bmatrix}\right\} + (-1)Max\left\{\begin{bmatrix}6 & 5\\4 & 7\end{bmatrix}\right\}$$

$$= 3 - 7 = -4$$

Hence, in general the operation is nonlinear

2. a)

The median of a sub image is a linear operation if and only if

$$Median[a_1 f_1(x,y) + a_2 f_2(x,y)] = a_1 Median[f_1(x,y)] + a_2 Median[f_2(x,y)] \qquad (1)$$

For any real constants a_1 and a_2, where $f_1$and f_2 are any two sub-images.

$$let\ f_1 = \begin{bmatrix}3 & 4 & 3\\4 & 8 & 7\\9 & 8 & 9\end{bmatrix}, and\ f_2 = \begin{bmatrix}2 & 2 & 0\\0 & 3 & 1\\2 & 0 & 0\end{bmatrix}$$

$$Median[f_1(x,y)] = 7, \quad Median[f_2(x,y)] = 1, let\ a_1 = 1\ and\ a_2 = -1$$

$$LHS\ of\ (1): a_1 Median[f_1(x,y)] + a_2 Median[f_2(x,y)] = 7 - 1 = 6$$

$$RHS\ of\ (1): Median[a_1 f_1(x,y) + a_2 f_2(x,y)] = Median\begin{bmatrix}1 & 2 & 3\\4 & 5 & 6\\7 & 8 & 9\end{bmatrix} = 5 \neq LHS$$

So, the median operation is not linear

b)

NO. A mask and correlation is used in linear filtering only and cannot be used to apply nonlinear operation

3. (a)

The average at any pair of coordinates (x,y) of a set of K images is given by

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i\,(x,y) = \frac{1}{K}\sum_{i=1}^{K} f_i\,(x,y) + \frac{1}{K}\sum_{i=1}^{K} \eta_i\,(x,y)$$

$$E\{\overline{g(x,y)}\} = E\left\{\frac{1}{K}\sum_{i=1}^{K} f_i\,(x,y) + \frac{1}{K}\sum_{i=1}^{K} \eta_i\,(x,y)\right\}$$

Applying that the expected value of the sum is the same as the sum of the expected values of the items included in the sum

$$E\{\overline{g(x,y)}\} = \frac{1}{K}\sum_{i=1}^{K} E\{f_i(x,y)\} + \frac{1}{K}\sum_{i=1}^{K} E\{\eta_i(x,y)\}$$

The expected value of a constant is the constant itself. This fact applies on $f_i(x,y)$ since the value of the original image is always the same; that is a constant. Another probability rule says that the expected value of a random variable of a given mean is the mean itself. This rule applies on $\eta_i(x,y)$ that has a zero mean as stated above. Hence,

$$E\{\overline{g(x,y)}\} = f(x,y)$$

(b)

Restarting from the equation

$$\bar{g}(x,y) = \frac{1}{K}\sum_{i=1}^{K} g_i\,(x,y) = \frac{1}{K}\sum_{i=1}^{K} f_i\,(x,y) + \frac{1}{K}\sum_{i=1}^{K} \eta_i\,(x,y)$$

It is known from random-variable theory that the variance of the sum of uncorrelated random variables is the sum of the variances of those variables. In addition, the variance of a constant value $f_i(x,y)$ is zero. Then, we can write

$$\sigma^2{}_{\overline{g(x,y)}} = \sigma^2{}_{f(x,y)} + \frac{1}{K^2}\left[\sigma^2{}_{\eta_1(x,y)} + \sigma^2{}_{\eta_2(x,y)} + \cdots + \sigma^2{}_{\eta_K(x,y)}\right]$$
$$= \frac{1}{K^2}\left[\sigma^2{}_{\eta_1(x,y)} + \sigma^2{}_{\eta_2(x,y)} + \cdots + \sigma^2{}_{\eta_K(x,y)}\right]$$

But η_i are samples of the noise η that has a single variance which is $\sigma^2{}_{\eta(x,y)}$. Hence,

$$\sigma^2{}_{\overline{g(x,y)}} = \frac{K}{K^2}\left(\sigma^2{}_{\eta(x,y)}\right) = \frac{1}{K}\sigma^2{}_{\eta(x,y)}$$

The relation of standard deviation is obtained by just taking the square root of both sides in the above equation.

$$\sigma_{\overline{g(x,y)}} = \frac{1}{\sqrt{K}}\sigma_{\eta(x,y)}$$

(c)

The condition that must be satisfied when practically applying the process is that the image to be averaged must be registered (aligned) in order to avoid the introduction of blurring and other artifacts in the output images

4. Let $g\,(x,y)$ denote the golden image, and let $f(x,y)$ denote any input image acquired during routine operation of the system. Change detection via subtraction is based on computing the simple difference $d(x,y) = g\,(x,y) - f\,(x,y)$. The resulting image, $d\,(x,y)$, can be used in

two fundamental ways for change detection.

One way is pixel-by-pixel analysis. In this case, we say that $f(x,y)$ is "close enough" to the golden image if all the pixels in $d(x,y)$ fall within a specified threshold band $[T_{min}, T_{max}]$ where T_{min} is negative and T_{max} is positive. Usually, the same value of threshold is used for both negative and positive differences, so that we have a band $[-T,T]$ in which all pixels of $d(x,y)$ must fall in order for $f(x,y)$ to be declared acceptable.

The second major approach is simply to sum all the pixels in $d(x,y)$ and compare the sum against a threshold Q. Note that the absolute value needs to be used to avoid errors canceling out. This is a much cruder test, so we will concentrate on the first approach.

There are three fundamental factors that need tight control for difference based inspection to work:

(1) Proper registration
(2) Controlled illumination
(3) Noise levels that are low enough so that difference values are not affected appreciably by variations due to noise.

The first condition basically addresses the requirement that comparisons be made between corresponding pixels. Two images can be identical, but if they are displaced with respect to each other, comparing the differences between them makes no sense. Often, special markings are manufactured into the product for mechanical or image-based alignment.

Controlled illumination (note that "illumination" is not limited to visible light) obviously is important because changes in illumination can affect dramatically the values in a difference image. One approach used often in conjunction with illumination control is intensity scaling based on actual conditions. For example, the products could have one or more small patches of a tightly controlled color, and the intensity (and perhaps even color) of each pixels in the entire image would be modified based on the actual versus expected intensity and/or color of the patches in the image being processed.

Finally, the noise content of a difference image needs to be low enough so that it does not materially affect comparisons between the golden and input images. Good signal strength goes a long way toward reducing the effects of noise. Another (sometimes complementary) approach is to implement image processing techniques (e.g., image averaging) to reduce noise.

Obviously there are a number if variations of the basic theme just described. For example, additional intelligence in the form of tests which are more sophisticated than pixel-by-pixel threshold comparisons can be implemented. A technique used often in this regard is to subdivide the golden image into different regions and perform different (usually more than one) tests in each of the regions, based on expected region content.

5. (a)

Pixels are integer values, and 8 bits allow representation of 256 contiguous integer values. In our work, the range of intensity values for 8-bit images is [0,255]. The subtraction of values in this range covers the range [−255,255]. This range of values cannot be covered by 8 bits, but it is given in the problem statement that the result of subtraction has to be represented in 8 bits also (0→255) and, consistent with the range of values used for 8-bit images, we assume that values of the 8-bit

الصفحة دى كانت حبة توضيح عالهامش

ـنـا احلم على عملية على النصا عملية في Linear او عمـ

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Comput

) Subtraction of image $b(x,y)$ from $a(x,y)$
l results values in the range [-255, 255]

2) this ranged Can not be Covered by 8 bit
image, so that ~~difference~~ resulting
ference quantities that ~~are~~ negative will be
Clipped at 0.

$$\boxed{d_k(x,y) = a(x,y) - \sum_{1}^{k} b(x,y)}$$

3) repeating subtraction (at most 255)
the difference will be:

① Zero: at location where $b(x,y)$ is not zero

② $a(x,y)$: at location where $b(x,y)$ is zero.

a b 5 1

41 - 1 = 3

-5

$a - \sum_{1}^{k} b$

d

a . b

difference images are in the range [0,255]. What this means is that any subtraction of 2 pixels that yields a negative quantity will be clipped at 0.

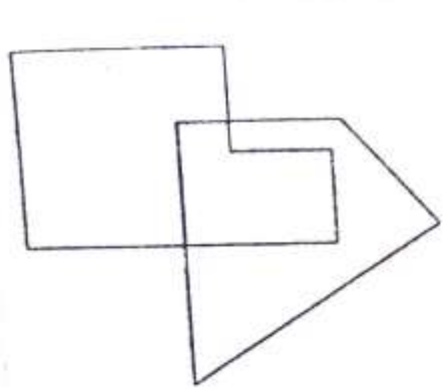
The process of repeated subtractions of an image $b(x,y)$ from an image $a(x,y)$ can be expressed as

$$d_k(x,y) = a(x,y) - \sum_{k=1}^{K} b(x,y)$$

$$a(x,y) - K \times b(x,y)$$

Where $d_K(x,y)$ is the difference image resulting after K subtractions. Because image subtraction is an array operation (see Section 2.6.1), we can focus attention on the subtraction of any corresponding pair of pixels in the images. We have already stated that negative results are clipped at 0. Once a 0 result is obtained, it will remain so because subtraction of any nonnegative value from 0 is a negative quantity which, again, is clipped at 0. Similarly, any location (x_0,y_0) for which $b(x_0,y_0)= 0$, will produce the result $dK(x_0,y0)= a(x_0,y_0)$. That is, repeatedly subtracting 0 from any value results in that value. The locations in $b(x,y)$ that are not 0 will eventually decrease the corresponding values in $d_K(x,y)$ until they are 0. The maximum number of subtractions in which this takes place in the context of the present problem is 255, which corresponds to the condition at a location in which $a(x,y)$ is 255 and $b(x,y)$ is 1. Thus, we conclude from the preceding discussion that repeatedly subtracting an image from another will result in a difference image whose components are 0 in the locations in $b(x,y)$ that are not zero and equal to the original values of $a(x,y)$ at the locations in $b(x,y)$ that are 0. This result will be achieved in, at most, 255 subtractions.

6. (a)



7. a) Noise reduction by averaging is an application of image addition, especially in astronomy. Images taken under very low light level frequently causes sensor noise render single image virtually useless for analysis. Averaging a number of images taken for the same scene under low light level reduces the noise contents in the average image. As the number of images averaged increases as the noise content in the resulting average image decreases.

b) Enhancement of differences between images is an application of image subtraction. Mask mode radiography is based on image subtraction. A mask h(x , y) is an X-ray image of a region of the patient's body. The procedure consists of injecting an X-ray contrast medium into the patient's bloodstream, taking a series of images called live images of the same region as that of h(x , y), and subtracting the mask from the series of incoming live images after injection of contrast medium. The net effect of subtracting the mask from each sample live image is that the areas under that are different between the live image and the mask appears in the output image enhanced.

mask mode radiography

mask mode
radio graphy